

Statistics for Medical Research (Part I)

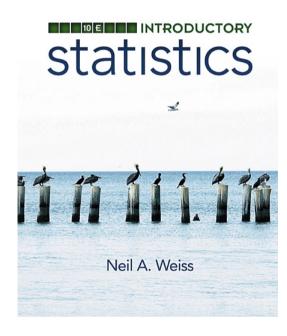
Descriptive Statistics Null Hypothesis Testing

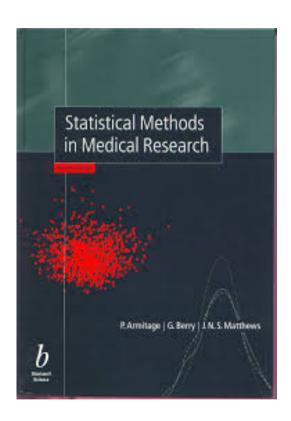
Dr. Wong Kai Choi

Department of Psychiatry
Pamela Youde Nethersole Eastern Hospital
19th January 2016











What is Statistics?



Mathematics and Statistics







Discovery of Fact







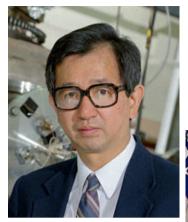


"Every block of stone has a statue inside it and it is the task of the sculptor to discover it." – Michelangelo

















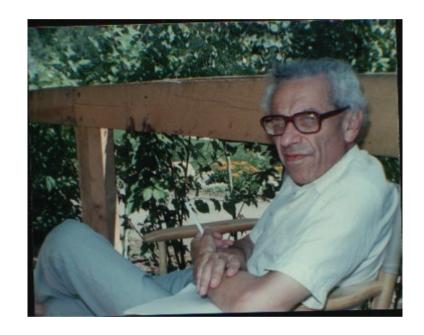






"A mathematician is a machine for turning coffee into theorems"

- Paul Erdos (1913 – 1996)





"It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment."

- Carl Friedrich Gauss (1777 – 1855)



Barl Friedrich Gauss.



"To call in the statistician after the experiment is done may be no more than asking him to perform a post-mortem examination: he may be able to say what the experiment died of"

- Sir R.A. Fisher Indian Statistical Congress (1938)





Descriptive Statistics



Parameter and Statistics

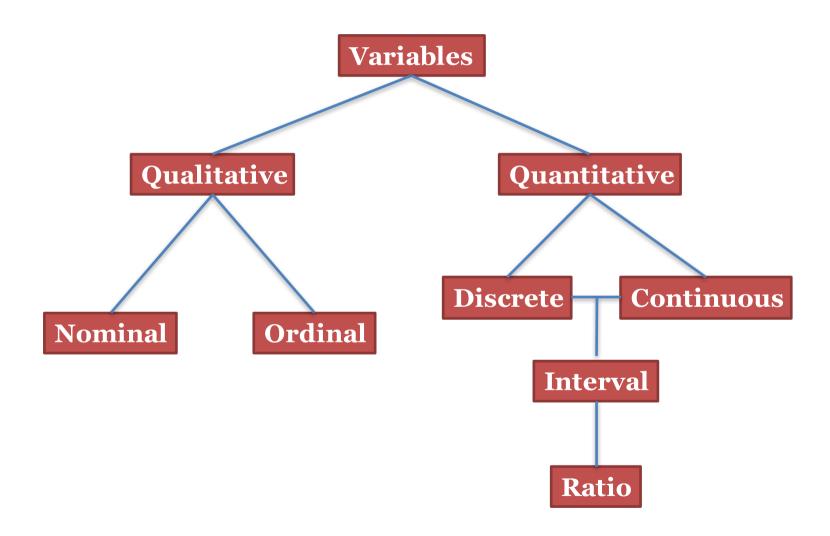
Parameter and Statistic

Parameter: A descriptive measure for a population

Statistic: A descriptive measure for a sample



Types of variables





Nominal Variables (類別變數) – Variables that have no numerical values

Ordinal Variables (有序變數) – values whose order is significant, but no meaningful arithmetic like operations can be performed.

Interval Variables (等距變數) – an ordinal variables that the same magnitudes of the differences between two values takes the same meaning.

Ratio Variables (比率變數) – with features of interval variables and any two values have meaningful ratio, making the operation of multiplication and division meaningful.



Descriptive Statistics

- It described the main features of a collection of data quantitatively
- It aimed at summarize the dataset
- There are 4 degrees:
 - Location
 - Spread
 - Skewness
 - Kurtosis



Measures of Central Location

- A measure of central location are used to pinpoint the center of a set of data values. It is often used to work out a typical value to represent the whole set of data.
- There are three commonly used measures of central location:

mean, mode and median.

The mean is the most common one to be used.

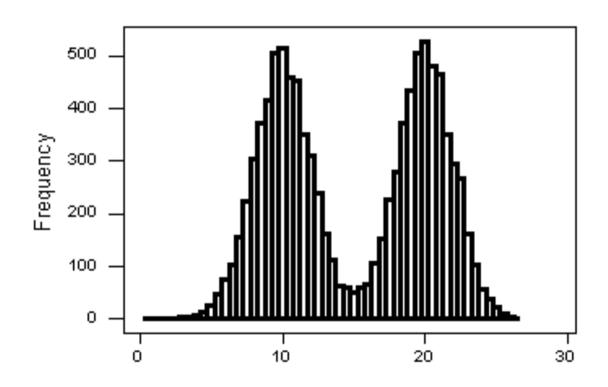


Location

- It is the first degree
- Mean: the arithmetic means or expected value of random variables
- Median: the value separating the higher half of a sample from the lower half
- Mode: The most common value among the group
- In symmetrical (not only normal) distribution: mean = median = mode



Symmetric, Double-peaked (Bimodal) Distribution





Spread

- It is a second degree
- Standard deviation: related to mean

$$\sigma = \sqrt{\mathrm{E}\left[(X - \mu)^2\right]}.$$

- Range and Percentile
- Interquartile range: related to median
 - It contain half of the sample inside the range
 - Upper quartile: separate the higher ¼ and lower ¾
 - Lower quartile: separate the lower ¼ and higher ¾



Measures of Variability

(Measures of Dispersion)

- Range
- Mean Absolute Deviation
- Variance and Standard Deviation
- Quartiles



The range

- -The range of a set of measurements is the difference between the largest and the smallest measurements.
- -Its major advantage is the ease with which it can be computed.
- -Its major shortcoming is its failure to provide information on the dispersion of the values between the two end points.



Quartiles

Quartiles

First, arrange the data in increasing order. Next, determine the median. Then, divide the (ordered) data set into two halves, a bottom half and a top half; if the number of observations is odd, include the median in both halves.

- •The **first quartile** (Q_1) is the median of the bottom half of the data set.
- •The **second quartile** (Q_2) is the median of the entire data set.
- •The **third quartile** (Q_3) is the median of the top half of the data set.



Mean absolute deviation

$$MAD = \frac{\sum_{i=1}^{N} |X_i - \mu|}{N}$$

Consider the data from a population set: 2, 7, 4, 12, 5.

$$MAD = \frac{|2-6| + |7-6| + |4-6| + |12-6| + |5-6|}{5}$$
$$= \frac{4+1+2+6+1}{5} = 2.8$$



The population variance

- This measure of dispersion reflects the values of *all* the measurements.
- The variance of a **population** of N measurements $x_1, x_2,...,x_N$ having a mean μ is defined as

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N}$$



The sample variance

- The variance of a **sample** of n measurements $x_1, x_2, ..., x_n$ having a mean \overline{X} is defined as

$$S^{2} = \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}}{n-1}$$

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Standard Deviation

The standard deviation of a set of measurements is the square root of the variance of the measurements.

Samplestandard deviation :
$$s = \sqrt{s^2}$$

Population standard deviation :
$$\sigma = \sqrt{\sigma^2}$$



Empirical Rules

Empirical Rule

For any quantitative data set with roughly a bell-shaped distribution, the following properties hold.

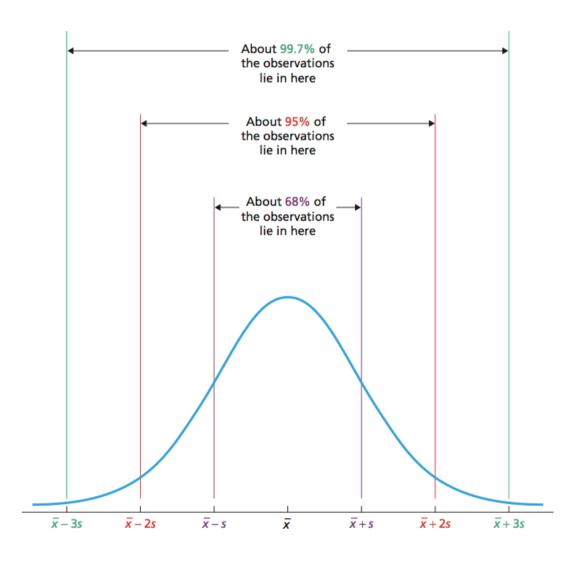
Property 1: Approximately 68% of the observations lie within one standard deviation to either side of the mean, that is, between $\bar{x} - s$ and $\bar{x} + s$.

Property 2: Approximately 95% of the observations lie within two standard deviations to either side of the mean, that is, between $\bar{x} - 2s$ and $\bar{x} + 2s$.

Property 3: Approximately 99.7% of the observations lie within three standard deviations to either side of the mean, that is, between $\bar{x} - 3s$ and $\bar{x} + 3s$.



Empirical Rules





Skewness

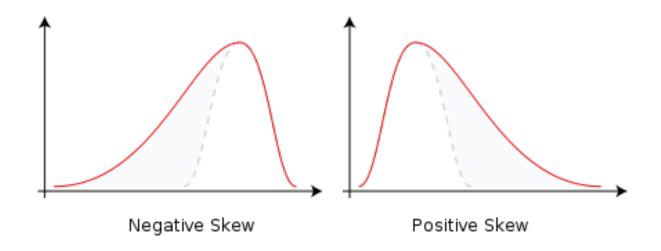
- It is the third degree
- It measure the asymmetry of the distribution
- It is calculated by

$$\gamma_1 = \mathrm{E}\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} = \frac{\mathrm{E}\left[(X-\mu)^3\right]}{(\mathrm{E}\left[(X-\mu)^2\right])^{3/2}} = \frac{\kappa_3}{\kappa_2^{3/2}},$$



Skewness

- Looked at the tail of the (unimodal) distribution
- Positive (right) skewed (mean > median > mode)
- Negative (left) skewed (mean < median < mode)

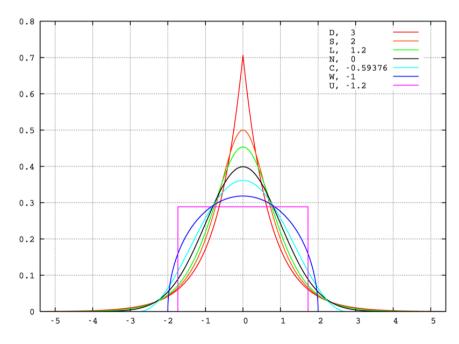




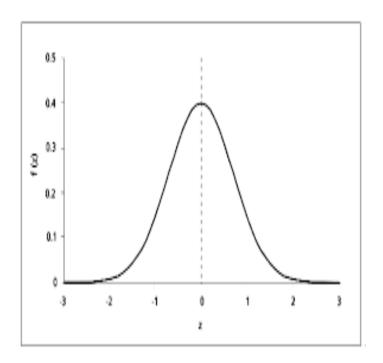
Kurtosis

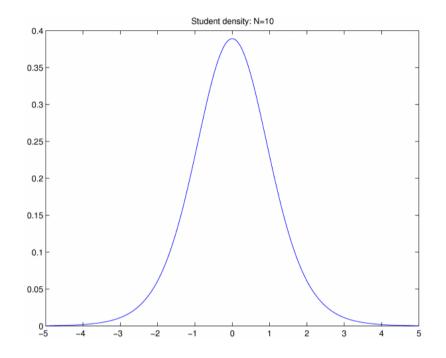
- It is the fourth degree
- It is a measure of "peakedness" of the distribution

$$g_2 = \frac{m_4}{m_2^2} - 3 = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \overline{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \overline{x})^2\right)^2} - 3$$









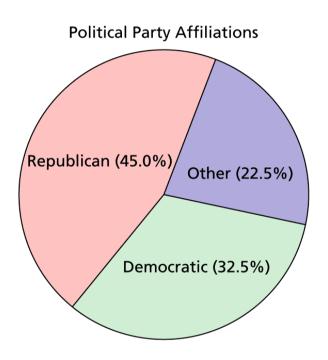
Normal Distribution

t-Distribution



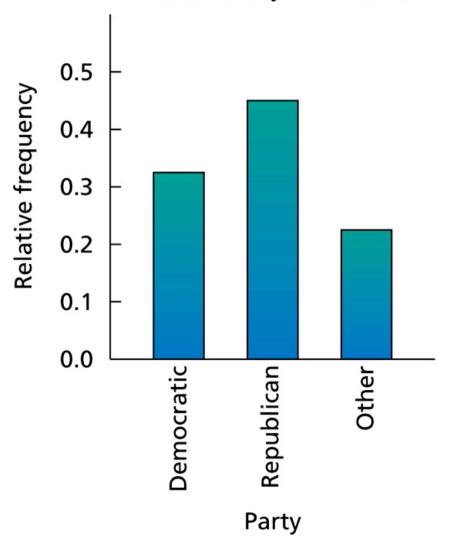
Organizing Qualitative Data







Political Party Affiliations

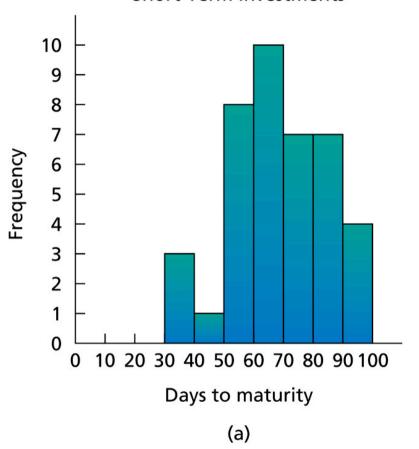


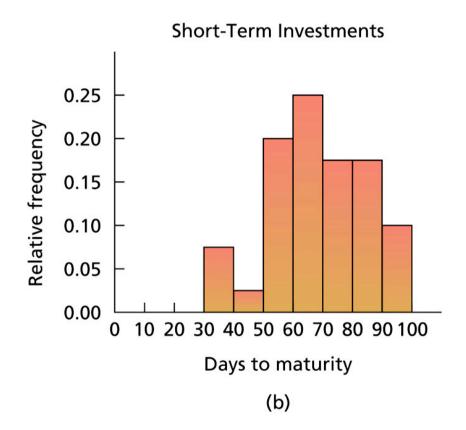


Organizing Quantitative Data



Short-Term Investments







Prices of DVD Players

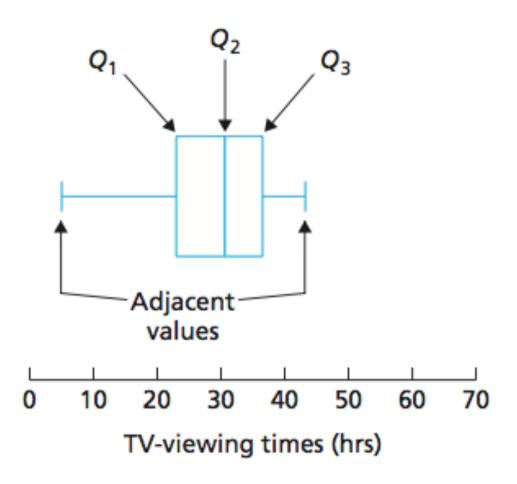




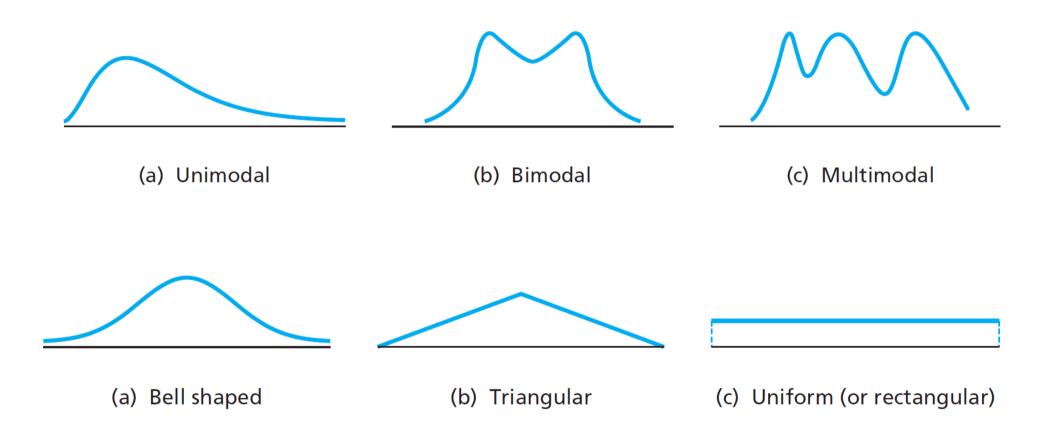
Stems Leaves (a) (b)



Boxplot









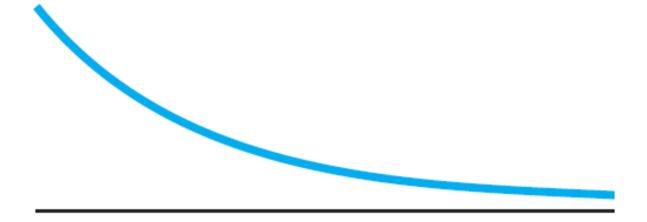


(a) Right skewed

(b) Left skewed



Reverse-J-shaped distribution





The Nature of Hypothesis Testing



Pioneers



Sir Ronald Aylmer Fisher (1890 – 1962)

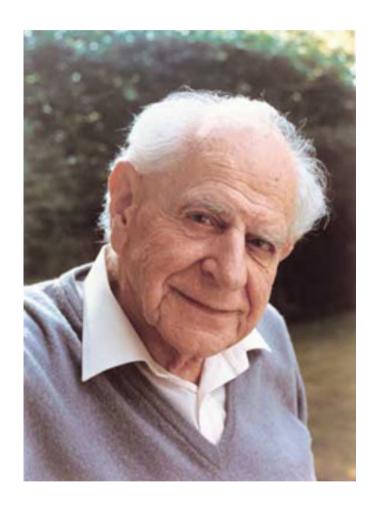


William Sealy Gosset (1876 - 1937)



Karl Pearson (1857 – 1936)

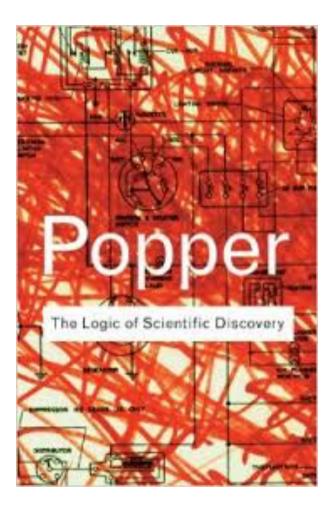




Sir Karl Raimund Popper (1902 – 1994)







The Logic of Scientific Discovery (1934)





Are all swans white?



Null and Alternative Hypotheses; Hypothesis Test

Null hypothesis: A hypothesis to be tested. We use the symbol H_0 to represent the null hypothesis.

Alternative hypothesis: A hypothesis to be considered as an alternative to the null hypothesis. We use the symbol H_a to represent the alternative hypothesis.

Hypothesis test: The problem in a hypothesis test is to decide whether the null hypothesis should be rejected in favor of the alternative hypothesis.



Hypothesis test do a simple job: *REJECT* or *NOT REJECT* the null hypothesis

Null hypothesis and Alternative hypothesis should be: *Mutually Exclusive Collectively Exhaustive*

Null Hypothesis should be *Negation* of Alternative Hypothesis



Null and Alternative Hypothesis For Unpaired Test

Null Hypothesis

 $\mu \leq \mu_0$

Alternative Hypothesis

 $\mu > \mu_0$

$$\mu = \mu_0 \qquad \qquad \mu \neq \mu_0 \\ \mu \geq \mu_0 \qquad \qquad \mu < \mu_0$$



Null and Alternative Hypothesis For Paired Test

Null Hypothesis

Alternative Hypothesis

$$\mu_1 - \mu_2 = 0$$



$$\mu_1 - \mu_2 \neq 0$$

$$\mu_1 - \mu_2 \ge 0$$



$$\mu_1 - \mu_2 < 0$$

$$\mu_1 - \mu_2 \le 0$$



$$\mu_1 - \mu_2 > 0$$



Type I and Type II Errors

Type I error: Rejecting the null hypothesis when it is in fact true.

Type II error: Not rejecting the null hypothesis when it is in fact false.



	H_0 is:			
		True	False	
Decision:	Do not reject H_0	Correct decision	Type II error	
	Reject H ₀	Type I error	Correct decision	
		CITOI	decision	



S.	Ho is True	Ho is False
Fail to Reject Ho	Innocence is correctly declared	Guilty person incorrectly set free
Reject Ho	Innocent person sent to prison	Guilt is correctly declared

		TRUTH ABOUT THE DEFENDENT	
		Innocent	Guilty
DECISION BASED ON TRIAL	Find Guilty	Type I Error	Correct decision
EVIDENCE	Find Not Guilty	Correct decision	Type II Error



Significance Level

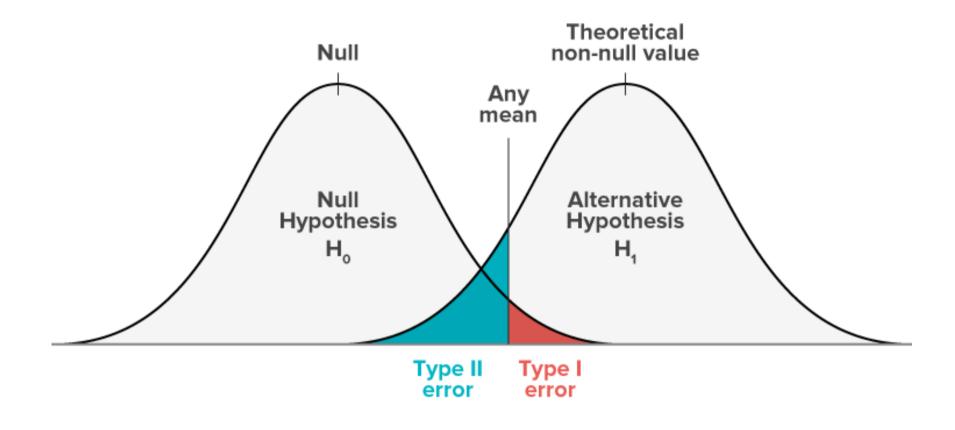
The probability of making a Type I error, that is, of rejecting a true null hypothesis, is called the **significance level,** α , of a hypothesis test.



Relation between Type I and Type II Error Probabilities

For a fixed sample size, the smaller we specify the significance level, α , the larger will be probability, β , of not rejecting a false null hypothesis.







Value of K corresponding to α and β

Dayyon (1 0)	Significance level (α)		
Power $(1 - \beta)$	5%	1%	0.1%
80%	7.8	11.7	17.1
90%	10.5	14.9	20.9
95%	13.0	17.8	24.3
99%	18.4	24.1	31.6

K is a value related to sample size. When K increases, sample size will increase



Possible Conclusions for a Hypothesis Test

Suppose that a hypothesis test is conducted at a small significance level.

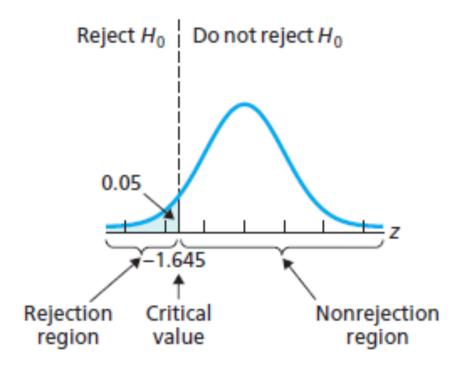
- If the null hypothesis is rejected, we conclude that the data provide sufficient evidence to support the alternative hypothesis.
- If the null hypothesis is not rejected, we conclude that the data do not provide sufficient evidence to support the alternative hypothesis.



Critical-Value Approach to Hypothesis Testing

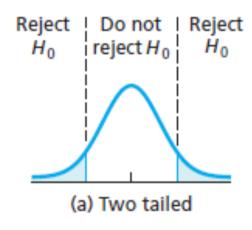


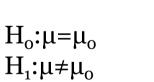
Criterion for deciding whether to reject the null hypothesis

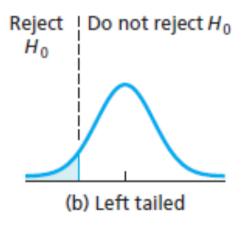


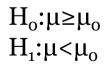


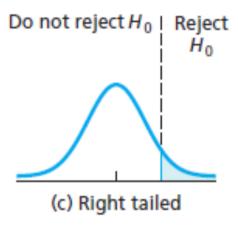
Graphical display of rejection regions for two-tailed, left-tailed, and right-tailed tests









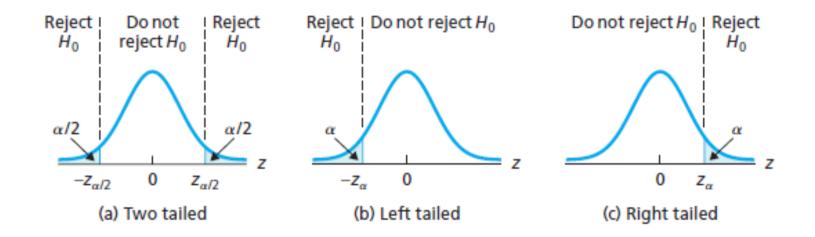


$$H_0: \mu \leq \mu_0$$

 $H_1: \mu > \mu_0$



Critical value(s) for a one-mean z-test at the significance level α if the test is (a) two tailed, (b) left tailed, or (c) right tailed





Z-value	$\mathbf{p} = 0.05$	$\mathbf{p} = 0.01$
Two sided test	1.959964	2.5758293
One-sided test	1.644854	2.3263479



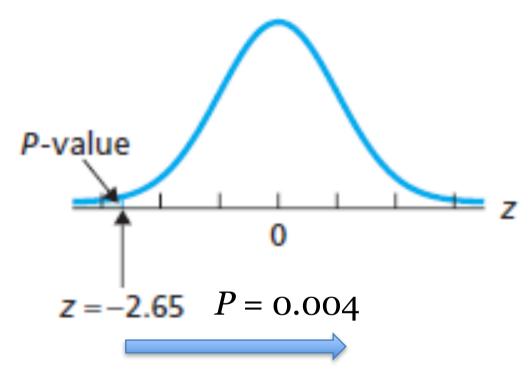
p-Value Approach to Hypothesis Testing



P-Value

The *P***-value** of a hypothesis test is the probability of getting sample data at least as inconsistent with the null hypothesis (and supportive of the alternative hypothesis) as the sample data actually obtained. We use the letter *P* to denote the *P*-value.





Null hypothesis can be rejected if significance level greater than p - value



General steps for the *P*-value approach to hypothesis testing

P-VALUE APPROACH TO HYPOTHESIS TESTING

- Step 1 State the null and alternative hypotheses.
- Step 2 Decide on the significance level, α .
- Step 3 Compute the value of the test statistic.
- Step 4 Determine the P-value, P.
- Step 5 If $P \le \alpha$, reject H_0 ; otherwise, do not reject H_0 .
- Step 6 Interpret the result of the hypothesis test.



Guidelines for using the *P*-value to assess the evidence against the null hypothesis

<i>P</i> -value	Evidence against H_0
$P > 0.10$ $0.05 < P \le 0.10$	Weak or none Moderate
$0.01 < P \le 0.05$ $P \le 0.01$	Strong Very strong



Hypothesis Tests Without Significance Levels:

Many researchers do not explicitly refer to significance levels. Instead, they simply obtain the *P*-value and use it (or let the reader use it) to assess the strength of the evidence against the null hypothesis.



Null hypothesis testing only answer a true or false question

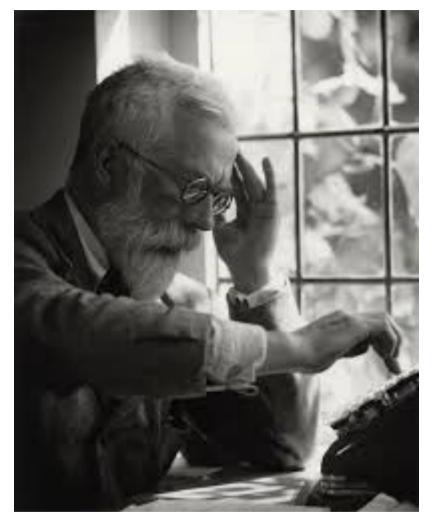
You must ask more than one "correct" questions to view a simple fact.



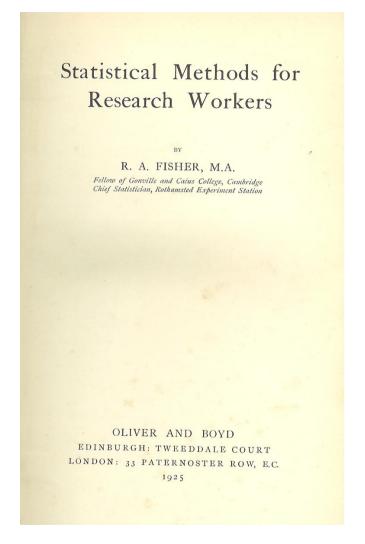


Significance level, conventionally, at 5%???





Sir Ronald Aylmer Fisher (1890 – 1962)



Statistical Method for Research Workers (1925)



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Tables I. and II. have been constructed to show the deviations corresponding to different values of this probability. The rapidity with which the probability falls off as the deviation increases is well shown in these tables. A deviation exceeding the standard deviation occurs about once in three trials. Twice the standard deviation is exceeded only about once in 22 trials, thrice the standard deviation

Statistical Methods for Research Workers (1925), p.44



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only once in 370 trials, while Table II. shows that to exceed the standard deviation sixfold would need nearly a thousand million trials. The value for which P = .05, or 1 in 20, is 1.96 or nearly 2; it is convenient to take this point as a limit in judging whether a deviation is to be considered significant or not. Deviations exceeding twice the standard deviation are thus formally regarded as significant.

Statistical Methods for Research Workers (1925), p.45

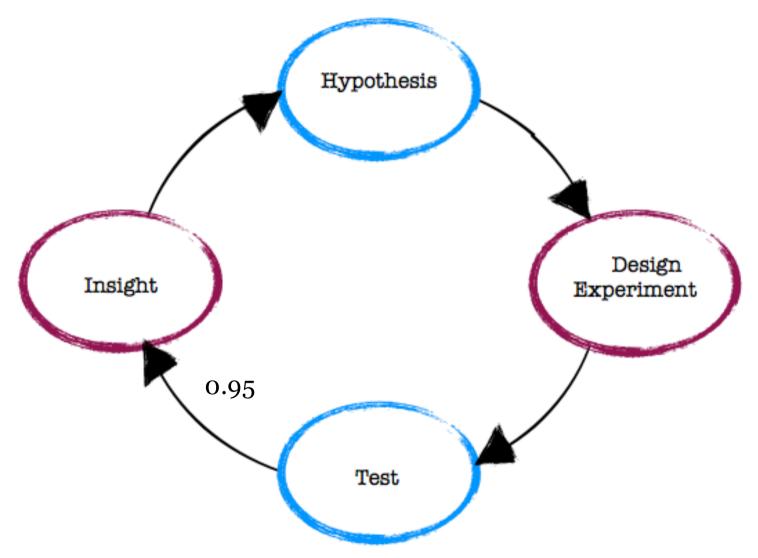


TABLE VI
TABLE OF z. P=·05

	Values of n ₁ .										
		I.	2.	3.	4.	5.	6.	8.	12.	24.	œ.
Values of n2.	I	2.5421	2.6479	2-6870	2.7071	2.1974	2.7276	2.7380	2.7484	2.7588	2.7693
	2	1.4592	1.4722	1-4765	1-4787	1.4800	1.4808	1.4819	1.4830	1.4840	1.485
	3	1.1577	1.1284	1.1137	1-1051	1.0994	1.0953	1.0899	1.0842	1.0781	1.071
	4	1.0212	-9690	-9429	-9272	-9168	-9093	-8993	-8885	-8767	-863
	5	.9441	-8777	-8441	-8236	-8097	.7997	.7862	.7714	.7550	.736
	6	-8948	-8188	.7798	.7558	.7394	.7274	.7112	-6931	-6729	-6499
	8	-8355	.7475	.7014	-6725	-6525	-6378	-6175	-5945	-5682	-537
	12	·7788	-6786	-6250	-5907	-5666	-5487	-5234	-4941	.4592	-4156
	24	-7246	-6123	.5508	-5106	-4817	.4598	·4283	-3904	*3425	-2749
	oc	-6729	·5486	-4787	.4319	.3974	-3706	-3309	-2804	•2085	0

VII. Interclass correlations and the Analysis of Variance







In Series

If
$$\alpha = 0.05$$

$$P$$
 (no Type I error after n cycles)
= $(1 - 0.05)^n$

When
$$n > 13, P < 0.5$$



In Parallel

If we repeat the same study for 20 times, there is one type I error, that is, wrongly reject the true null hypothesis.

Usually, the result will be published because it is "significant", "new unusual finding" and "eyeball-catching".







Editorial

David Trafimow and Michael Marks

New Mexico State University

The *Basic and Applied Social Psychology* (BASP) 2014 Editorial emphasized that the null hypothesis significance testing procedure (NHSTP) is invalid, and thus authors would be not required to perform it (Trafimow, 2014). However, to allow authors a grace period, the Editorial stopped short of actually banning the NHSTP. The purpose of the present Editorial is to announce that the grace period is over. From now on, BASP is banning the NHSTP.

Basic and Applied Social Psychology, 37: 1-2, 12 Feb 2015



SOCIAL SELECTION

Popular articles on social media

Psychology journal bans Pvalues

A controversial statistical test has met its end, at least in one journal. Earlier this month, the editors of Basic and Applied Social Psychology (BASP) announced that the journal would no longer publish papers containing P values, because the values were too often used to support lower-quality research.

Authors are still free to submit papers to BASP with P values and other statistical measures that form part of 'null hypothesis significance testing' (NHST), but the numbers will be removed before publication. "Basic and Applied Social Psychology just went science rogue and banned NHST from their journal. Awesome," tweeted Nerisa Dozo, a PhD student in psychology at the University of Queensland in Brisbane, Australia. But Jan de Ruiter, a cognitive scientist at Bielefeld University in Germany, tweeted: "NHST is really problematic", adding that banning all inferential statistics is "throwing away the baby with the p-value".

Basic Appl. Soc. Psych. 37, 1-2 (2015)

Nature (26th February 2015)





Academic journal bans p-value significance test

Written by Web News Editor on 05 March 2015. Posted in News



An editorial published in the academic journal *Basic and Applied Social Psychology* (BASP) has declared that the null hypothesis significance testing procedure (NHSTP) is 'invalid', and have banned it from future papers submitted to the journal.

In significance testing, a null hypothesis typically posits that changing the input to a system does not change the resulting output. To determine whether a result is statistically significant, a researcher has to calculate a p-value, which is the probability of observing an apparent effect given that the null hypothesis is true. If the p-value is less than 0.05 it is conventionally deemed a statistically significant result.

'We believe that the p < 0.05 bar is too easy to pass and sometimes serves as an excuse for lower quality research,' BASP authors David Trafimow and Michael Marks of Mexico State University write in their editorial. They go on to say that they hope other journals will follow suit.

Confidence intervals are also banned, since they 'do not provide a strong case for concluding that the population parameter of interest is likely to be within the stated interval.' Bayesian alternatives, however, 'are neither required nor banned' from the journal.

The use of p-values to judge whether research is 'significant' has been thought by many statisticians to be problematic, ever since the idea was introduced by Ronald Fisher in the 1920s. In 2005, Stanford School of Medicine professor John loannidis wrote a well-known paper, 'Why Most Published Research Findings Are False', stating that reliance on p-values alone was 'ill-founded'.

More recently, in November 2014, Royal Society Fellow David Colquboun published 'An investigation of the false discovery rate and the misinterpretation of p-values' stating: 'If you use p=0.05 to suggest that you have made a discovery, you will be wrong at least 30% of the time'.

The American Statistical Association (ASA) argues that banning p-values altogether will have other 'negative consequences' in a formal statement regarding the editorial. 'The statistical community is aware of problems associated with the use and interpretation of inferential methods,' it says. 'However, the journal proposes to fall back entirely on descriptive statistics and use "larger sample sizes than is typical in much psychology research." We believe this policy may have its own negative consequences and thus the proper use of inferential methods needs to be analyzed and debated in the larger research community.'

RSS president, Peter Diggle, said: 'The RSS welcomes and shares the BASP editors' concerns that inferential statistical methods are open to misuse and mis-interpretation, but does not feel that a blanket ban on any particular inferential method is the most constructive response.'

StatsLife (5th March 2015)



In November 2014, Royal Society Fellow

David Colquhoun published

'An investigation of the false discovery rate
and the misinterpretation of p-values' stating:

'If you use p=0.05 to suggest that you
have made a discovery, you will be
wrong at least 30% of the time'.



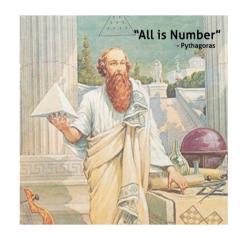
Evidence-Based medicine relies on research with 5% significance level Fundamentals of Evidence-Based Medicine



5% Significance Level itself is not Evidence Based



Three Mathematical Crises



5th century BC



17th century



20th century



Frist Statistical Crisis

The Statistical Crisis in Science

Data-dependent analysis—a "garden of forking paths"— explains why many statistically significant comparisons don't hold up.

Andrew Gelman and Eric Loken

here is a growing realization that reported "statistically significant" claims in scientific publications are routinely mistaken. Researchers typically express the confidence in their data in terms of *p*-value: the probability that a perceived result is actually the result of random variation. The value of *p* (for "probability") is a way of measuring

a short mathematics test when it is expressed in two different contexts, involving either healthcare or the military. The question may be framed nonspecifically as an investigation of possible associations between party affiliation and mathematical reasoning across contexts. The null hypothesis is that the political context is irrelevant to the task, and the alternative hypothesis.

This *multiple comparisons* issue is well known in statistics and has been called "*p*-hacking" in an influential 2011 paper by the psychology researchers Joseph Simmons, Leif Nelson, and Uri Simonsohn. Our main point in the present article is that it is possible to have multiple potential comparisons (that is, a data analysis whose details are highly contingent

American Scientist (Nov-Dec 2014)







If you use p = 0.05 to suggest that you have made a discovery, you will be wrong at least 30% of the time. If, as is often the case, experiments are underpowered, you will be wrong most of the time. This conclusion is demonstrated from several points of view. First, tree diagrams which show the close analogy with the screening test problem. Similar conclusions are drawn by repeated simulations of *t*-tests. These mimic what is done in real life, which makes the results more persuasive. The simulation method is used also to evaluate the extent to which effect sizes are over-estimated, especially in underpowered experiments. A script is supplied to allow the reader to do simulations themselves, with numbers appropriate for their own work. It is concluded that if you wish to keep your false discovery rate below 5%, you need to use a three-sigma rule, or to insist on $p \le 0.001$. And *never* use the word 'significant'.

David Colquhoun 'An investigation of the false discovery rate and the misinterpretation of p-values', **Royal Society Open Science**, 19th November 2014







