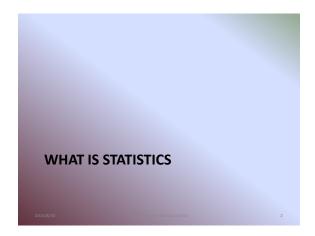
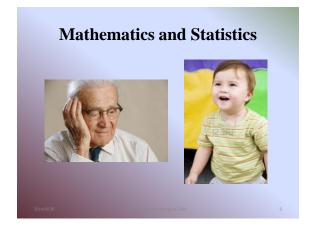
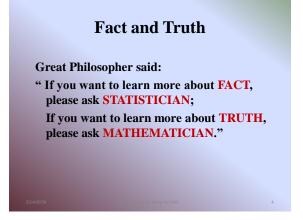
## Training Course on Research & Biostatistics for Medical Professionals For Hong Kong College of Family Physicians Dr. Wong Kai Choi MBChB, BSc, MEd, GradStat, AMIMA, FHKAM(Psychiatry), FHKCPsych 30th August 2014







### Role of Medical Statistician

- Should be involved in the beginning of study
- Advise on study design
- Calculation of sample size (if appropriate)
- Choice of appropriate statistical test
- Data analysis
- Data management
- Interpretation of statistical result

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"To call in the statistician after the experiment is done may be no more than asking him to perform a post-mortem examination: he may be able to say what the experiment died of"

- Sir R.A. Fisher Indian Statistical Congress (1938)



- Sample size calculation · Sampling method
- Descriptive statistics
- Type of data and data management
- Statistical test and significant test
- Interpretation of statistical result
- Common statistical software

### **SAMPLE SIZE CALCULATION**

### **Sample**

- Subset of population
- Rarely use the entire population
- May be imperfect representation of population
- If sample is unbiased and large enough, it will give useful information

### If sample size too small

- 95% confidence interval will not be narrow enough to give precise estimate (mean, median or proportion)
- In a comparative test, a true difference may be missed
- Unethical to the subject involved and the society (if the intervention is beneficial with minimal side effect)

### If the sample size too large

- Wasting of resource, including time, money, manpower, etc.
- In naturalistic study, positive result (in comparative test) with low clinical significant will be detected.
- · Unethical to society and subject (if intervention with no real effect but have serious side effect)

### Sample size for means estimation

- · Information required
  - Standard deviation (SD) of the measure being estimated (can be from previous studies)
  - The desired width of the confidence interval (d)
  - The confidence interval (CI) (95%, 99%, 90%, etc)

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### Sample size for means estimation

- $n = (CI)^2 \times 4 (SD)^2 / d^2$ 
  - For 95% confidence interval, CI = 1.96
  - For 90% confidence interval, CI = 1.64
  - For 99% confidence interval, CI = 2.58

8/30

2014/8/30

### Sample size for proportion estimation

- · Information required
  - The expected population proportion, p
  - The desired width of the confidence interval (d)
  - The confidence interval (CI)

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### Sample size for proportion estimation

- $n = (CI)^2 \times 4 p(1-p)/d^2$ 
  - For 95% confidence interval, CI = 1.96
  - For 90% confidence interval, CI = 1.64
  - For 99% confidence interval, CI = 2.58

4/8/30

### Sample size for comparative studies

- Sample size calculation aim at minimize the type I and type II error
- Type I error we conclude that there is a difference between groups in the target population when in fact there is not.
- Type II error We conclude that there is no difference between the groups in the target population when in fact there is a real difference.

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### Sample size for comparative studies

- Clinically important difference the minimum clinically important difference is determined by physician who would not want the study miss it. It should be clinically meaningful but not statistically meaningful
- Significance level (α) it is the probability of type I error
- The power of the test  $(1 \beta)$  where  $\beta$  is the probability of type II error

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### Sample size for comparing means

- The information required:
  - The standard deviation (SD) of the measure being compared
  - The minimum difference (d) that is clinically important
  - The significance level ( $\alpha$ )
  - The power of the test  $(1 \beta)$
  - -n is the number of sample in each group
- $n = 2K(SD)^2 / d^2$

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### Sample size for comparing proportions

- The information required:
  - The expected population proportions in group 1, P<sub>1</sub>
  - The expected population proportions in group 2, P<sub>2</sub>
  - The significance level  $(\alpha)$
  - The power of the test  $(1 \beta)$
  - -n is the number of sample in each group
- $n = K[P_1(1-P_1) + P_2(1-P_2)]/(P_1-P_2)^2$

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### Value of K corresponding to $\alpha$ and $\beta$

D (1 0)		Significance level (α)	
Power (1 – β)	5%	1%	0.1%
80%	7.8	11.7	17.1
90%	10.5	14.9	20.9
95%	13.0	17.8	24.3
99%	18.4	24.1	31.6

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### **Assumption for sample size calculation**

- There was no sample lost till the end of study once the samples was selected.
- There are equal sample size in each group of comparison
- Samples are simple random sample selected at individual level
- It is not applied to cluster sampling and regression
- The samples are large enough (> 50 in each group) to use large sample methods for analysis

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### Other issues

- We can select more sample if we expected that there are sample lost during study
- Software can be used to calculate sample size
- Is calculation always needed:
  - Not if the study is a qualitative study
  - Not always for a small survey
  - Not always for a pilot study

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### SAMPLING METHODS

### Different types of sampling

- · Simple random sampling
- · Systematic sampling
- Stratified sampling
- Cluster sampling
- · Quota sampling
- Convenience sampling

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### **Simple Random Sampling**

- You should have whole list of subjects in sample space
- Label each subject with a number
- If you want n sample from sample space, select n random number form computer or random number table
- Select the sample label with the random number selected.

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### **Simple Random Sampling**

- Advantage
  - Gold standard of sampling
  - No detail of each sample is needed
  - Each individual has equal chance of being selected
- Disadvantage
  - You need to know the whole list in sampling frame

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### **Systematic sampling**

- We got whole list in sampling frame and label the subject by natural number
- Calculate interval I = population size / sample size
- Randomly choose a number less than I from computer program or random number table and we select the first sample which label by the random number selected
- Then, select another sample by interval *I* till the end of the list.

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### **Systematic sampling**

- Advantage
  - Easy to perform
  - If the list ranked in a group followed by another, it got the effect of stratified sampling
- Disadvantage
  - Cause sampling error if the list order in particular way (say, there must be a man followed by a woman)

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### **Stratified Sampling**

- Put each subject into one of the strata in which each stratum must be mutually exclusive and all strata must include all subject.
- The characteristics of subjects in particular stratum should be homogenous.
- From each stratum, select a proportion of sample by simple random sampling or systematic sampling and form subsample (the proportion = sample size / population size)
- · Combined all subsample.

8/30

### **Stratified Sampling**

- Advantage
  - We will not miss sample from minority group
  - Minimal sampling error
- Disadvantage
  - Time and manpower consuming
  - Some characteristic of each individual must be known
  - Resulting sample size may be larger than expected if there are too many strata, or there are some strata with few subjects only

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### **Cluster Sampling**

- Divided the population into several clusters (the difference in characteristics of subjects in each cluster should be minimal)
- · Randomly selected few clusters
- From each cluster, collect the list of sampling frame and selected certain number of sample from each cluster by simple random sampling, systematic sampling or stratified sampling.
- Combine all subsample

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**Cluster Sampling** 

- Advantage
  - Whole list of the population is not necessary
  - Decreased traffic time
- Disadvantage
  - Increased sampling error
  - Different method of sample size calculation

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**Quota Sampling** 

- Each interviewer assigned certain number of sample to be interviewed (location of selecting the sample may also be fixed)
- Certain proportion of characteristics are required (say, half should be female)
- If the number of sample achieved, the sampling process ended
- There was no randomization

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### **Convenience Sampling**

- · There was no limitation
- You can select any sample you like and agree to be interviewed or join the test
- Sampling error
- Convenience sampling should be avoided if possible

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### **Randomization**

- Randomization reduces bias by equalising socalled factors (independent variables) that have not been accounted for in the experimental design
- It can be achieved by random number table, or generated by computer (like Excel)
- Pseudo-random number generated by computer

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### **Descriptive Statistics**

- It described the main features of a collection of data quantitatively
- · It aimed at summarize the dataset
- There are 4 degrees:
  - Location
  - Spread
  - Skewness
  - Kurtosis

### **Spread**

- · It is a second degree
- · Standard deviation: related to mean

$$\sigma = \sqrt{\mathbb{E}[(X - \mu)^2]}.$$

- · Range and Percentile
- Interquartile range: related to median
  - It contain half of the sample inside the range
  - Upper quartile: separate the higher ¼ and lower ¾
  - Lower quartile: separate the lower ¼ and higher ¾

• It is the first degree

of random variables

a sample from the lower half

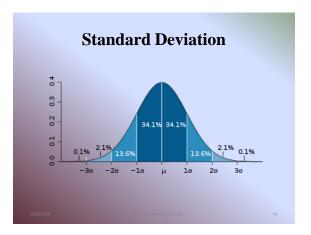
• In normal distribution: mean = median = mode

• Mode: The most common value among the

Location

• Mean: the arithmetic means or expected value

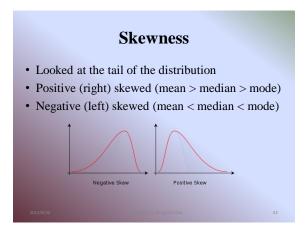
• Median: the value separating the higher half of

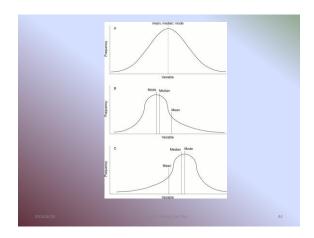


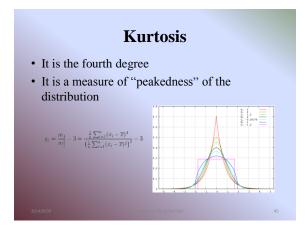
### **Skewness**

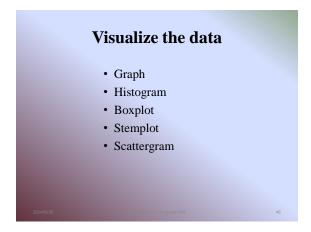
- It is the third degree
- It measure the asymmetry of the distribution
- It is calculated by

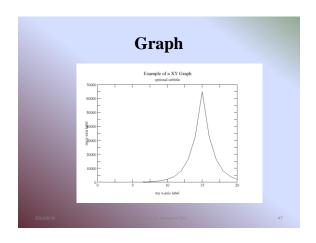
$$\gamma_1 = \mathrm{E} \Big[ \Big( \frac{X - \mu}{\sigma} \Big)^3 \, \Big] = \frac{\mu_3}{\sigma^3} = \frac{-\mathrm{E} \big[ (X - \mu)^3 \big]}{(\mathrm{E} \big[ (X - \mu)^2 \big])^{3/2}} = \frac{\kappa_3}{\kappa_2^{3/2}} \; ,$$

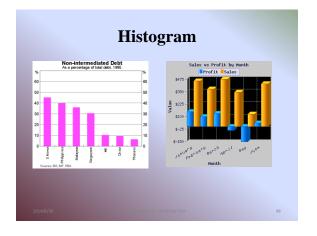


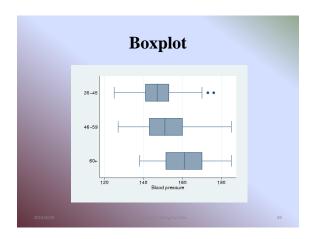


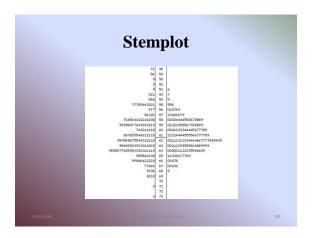


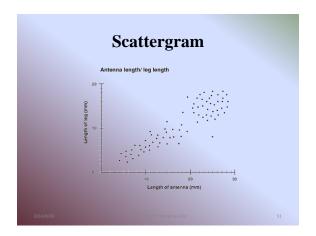


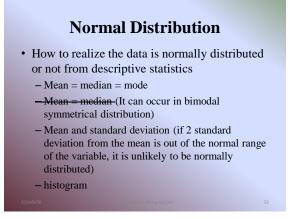














### Types of Data Either quantitative or categorical A variable is a quantity that is measured or observed in an individual and which varies from person to person A statistics (test statistics) is any quantity that is calculated from a set of data

### Quantitative data

- Can be measured numerically
- Continuous data lie on a continuum and can take any value between two limits
- Discrete data can take certain value (count)
- Interval scale differences between values at different points of the scale have the same meaning
- Ratio scale ratio of two measures has same meaning (temperature scale)
- Ordinal data data values can be arranged in a numerical order from the smallest to the largest.

### Categorical data

- Individuals fall into a number of separate categories or classes
- Number was assigned for coding purpose, it can be ordinal but not interval data because the "distance" has no real measure
- Distinguish between ordered and non-ordered data
- Dichotomous data only 2 classes
- · Categorizing continuous data
  - Dichotomizing potentially very problematic
  - Into several groups less effect on statistical power when compared with dichotomizing

### **Data Entry**

- Give a unique identifier to each subject
- Code the data before entry (remember the meaning of the code)
- Excel or statistical program
- · Can be directly transfer form electronic data
- Double entry can reduce the rate of error during data transfer

### Data storing and transporting

- Paper forms
  - File systematically to allow easy access
  - Stored securely
  - Store identifying detail separately from data form
  - Keeping a copy in one location if they are being transported
  - If data are valuable, a copy should always be kept in a different place.

### **Data storing and transporting**

- Electronic files
  - Identifying details should always be removed when transporting the file
  - Password protected and encrypted
  - Keep more than one copy of the data in two separate locations
  - Useful to use file names that show the version or date where files are updated

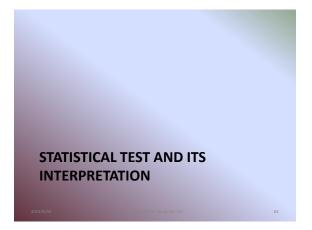
### **Data Checking**

- · Check early
- · Range checks
- Consistency
- · Original forms
- · Missing data
- Snowballing errors
- Digit preference (round up problem)
- · Scatter plot
- · Statistical analysis

### **Correcting errors**

- Check the original data form wherever possible to identify the source of potential data error
- Not to make assumptions or guesses where data values look unusual or are missing
- An outlying value should not be deleted simply because it is unusual.
- Keep a record of any changes that are made to the dataset and keep dated copies of datasets as changes are made.

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### **Hypothesis test**

### **Hypothesis testing**

- Set up hypothesis
- Find value of test statistics
- · Look up critical value
- Is test statistics smaller (or greater) than critical value
- Decide reject the hypothesis or not

2014/8/3

### Hypothesis test

- We decide that we should "reject" the hypothesis or not.
- If we want to know whether A is true
- We set a null hypothesis A
- Then, by means of rejecting null hypothesis to prove A is true
- · Why?

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### Hypothesis test

- We cannot check all the case in the world to prove the hypothesis is true
- But once there is a contra-example, we can disprove a statement or hypothesis
- So, it is easier to reject a null hypothesis rather than test all the case in the world to check a hypothesis
- It is the concept of Falsifiability or Refutability.

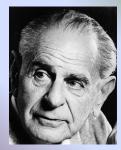
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### **Falsifiability**

A theory can be contradicted by an observation or the outcome of a physical experiment. That something is "falsifiable" does not mean it is false; rather, that if it is false, then some observation or experiment will produce a reproducible result that is in conflict with it.

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- Sir Karl Raimund Popper (1902 – 1994)
  From 1930 to 1936, he taught secondary school
- From 1930 to 1936, he taught secondary school. Popper published his first book, *Logik der Forschung (The Logic of Scientific Discovery)* in 1934, in which he introduce the concept of Falsifiability.



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### **Null / Alternative Hypothesis**

- The null hypothesis typically corresponds to a general or default position, that are capable of being proven false using a test of observed data.
- It is typically paired with a second hypothesis, the alternative hypothesis, which asserts a particular relationship between the phenomena.
- It is important to understand that the *null* hypothesis can never be proven. Your data can
   only reject a null hypothesis or fail to reject it.

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### Hypothesis of one sided test

 $H_0: M_A \leq M_B$ 

 $H_I: M_A > M_B$ 

Where  $H_0$  is null hypothesis and  $H_1$  is alternative hypothesis

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### Hypothesis of one sided paired test

 $H_0: M_{A-B} \leq 0$ 

 $H_1: M_{A-B} > 0$ 

Where  $H_0$  is null hypothesis and  $H_1$  is alternative hypothesis

### Hypothesis of two sided test

 $H_0: M_A = M_B$ 

 $H_1: M_A \neq M_B$ 

Where  $H_0$  is null hypothesis and  $H_1$  is alternative hypothesis

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### Hypothesis of two sided paired test

 $H_0: M_{A-B} = 0$  $H_1: M_{A-B} \neq 0$ 

Where  $H_0$  is null hypothesis and  $H_1$  is alternative hypothesis

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### Reject the hypothesis

- The hypothesis is rejected if a sample is selected whose values are one of the 5% most extreme outcomes that might occur if the hypothesis were true.
- In case of one way testing, 5% in one side
- In case of two way testing, 2.5% in each side.

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### P value

- P value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true
- P < 0.05 is usually regarded as statistically significant</li>
- Not significant does not mean "there is no difference" or "there is no effect". It means there is insufficient evidence for a difference or effect
- Exact p values should be given with estimates and confidence intervals wherever possible.

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**History of Statistics** 

- 1532 First weekly data on deaths in London (Sir W. Petty)
- 1539 start of data collection on birth, marriages and deaths in France
- 1662 First published demographic study based on bills of mortality (J. Braunt)
- Publication of Ars Conjectandi (J Bernoulli)
- 1834 establishment of Royal Statistical Society

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- 1839 Establishment of American Statistical Association (Boston)
- 1889 Publication of *Natural Inheritance* (F. Galton)
- 1900 development of chi-squared test (K Pearson)
- 1901 publication of first issue of *Biometrika* (F. Galton)
- 1903 development of Principal Component Analysis (K Pearson)

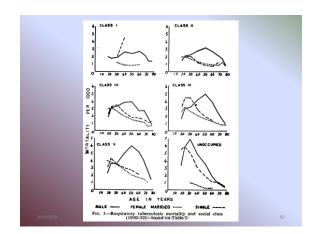
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- 1908 publication of *The Probable error of a mean* ("Student")
- 1920 Pearson create the role of medical statistician
- 1925 Publication of *Statistical Methods for Research Worker* (R A Fisher)
- 1935 Publication of The Design of Experiments (R A Fisher)

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- 1946 first clinical trial conducted by British Medical Research Council
- 1972 Publication of *Regression models and life tables* (D R Cox)
- 1979 Publication of *Bootstrap methods:* another look at the jackknife (B Efron)

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Drawbacks of using significant test in medical research

### Can Statistical Results apply to clinical setting?

Can collective phenomena explain individual behavior?



- Macroscopically, the survival rate of a particular disease is n%
  Microscopically, if a patient survives, survival rate is 100%; if patient dies, survival rate is 0% all or none
- Use of multiple regression analysis can partially solve the problem and try to individualize the treatment.

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Deduction vs Induction

### **Deductive Reasoning**

Deductive logic, is reasoning which constructs or evaluates deductive arguments. Deductive arguments are attempts to show that a conclusion necessarily follows from a set of premises. A deductive argument is valid if the conclusion must be true provided that the premises are true. A deductive argument is sound if it is valid and its premises are true.

2014/6

Deductive Reasoning

Theory

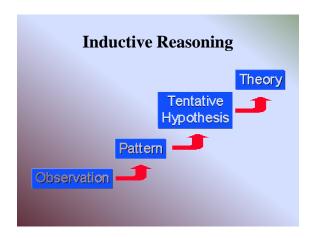
Hypothesis

Observation

Confirmation

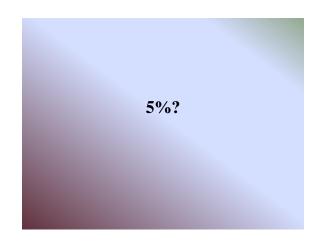
### **Inductive Reasoning**

Inductive logic, is a kind of reasoning that constructs or evaluates propositions that are abstractions of observations. It is commonly construed as a form of reasoning that makes generalizations based on individual instances. In this sense it is often contrasted with deductive reasoning

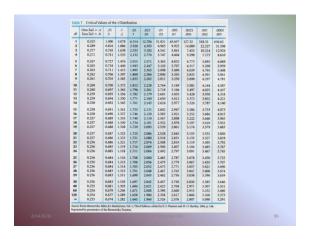


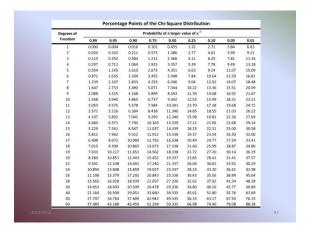
### Clinical Inductive / Deductive Reasoning Inductive Reasoning – symptoms of a patient is ..., and we draw a list of differential diagnoses Deductive Reasoning – we have a particular diagnosis in mind and compare the symptoms of the patient to see whether he fit the diagnosis or not.

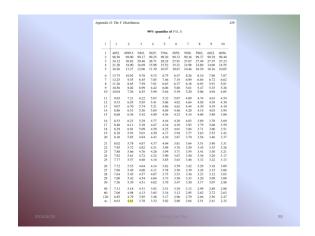
## Deductive Reasoning We can only acquire part of the truth with deductive reasoning, depends on the hypothesis we set. Example: investigator can only prove there is tail or a head in coin



## Why 5% is chosen? Fisher played a major role in the canonization of the 5% level as a criterion for statistical significance. In 1925, in his book "Statistical Methods for Research Workers" he fixed 5% as the only significance level in Table VI (F-distribution). 5% is arbitrary, as Fisher knew, but fulfils a general social purpose.



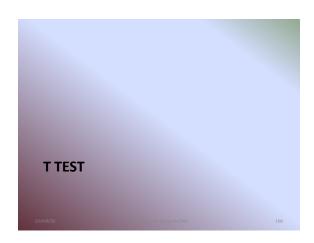




### 5% - evidence based?

- The value is fixed in 1925 when the medical statistics and clinical trial is not well established.
- 5% is reasonable for social research, as Fisher knew, but it may not reasonable in medical research
- Before we have evidence to show we should choose 5% as sigificiant level, we cannot say that we are practicing "evidence based medicine"

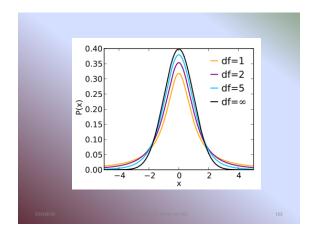
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### History

- The t-statistic was introduced in 1908 by William Sealy Gosset, a chemist working in Dublin, Ireland ("Student" was his pen name).
- He published the test in *Biometrika* in 1908

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### T-test for 2 independent means

- · Details of the test
  - -Compares means from 2 independent sample
  - Based on sampling distribution of difference of two samples
  - Allow calculate a difference and confidence interval of the difference
  - Can be calculated by formula or statistical program

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### T-test for 2 independent means

- · Null hypothesis
  - -Two samples come from population with same means
- Assumptions of test
  - -Continuous data with normal distribution
  - Variances are the same

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### T-test for 2 independent means

- · If assumptions do not hold
  - The statistical test is dubious and the p value may be wrong
  - Try transformation of the data
  - It is robust to slight skewness (2 samples with same size) but is less robust if variances are clearly different
  - Skewness and different variance can be corrected by transformation.

 $t = \frac{\overline{X}_1 - \overline{X}_2}{s_{\overline{X}_1 - \overline{X}_2}}$ 

Where

$$s_{\overline{X}_1 - \overline{X}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.$$

Degree of freedom

$$\mathrm{d.f.} = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1-1) + (s_2^2/n_2)^2/(n_2-1)}.$$

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### T test for paired (matched) data

- · Also called one sample t-test
- It analyses mean difference in paired sample
- Null hypothesis: means difference is zero
- Assumption
  - -differences follow a normal distribution
  - -variance are constant
- If assumption do not hold transform the raw data (not the difference)

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$$t = \frac{\overline{X}_D - \mu_0}{s_D / \sqrt{n}}.$$

Where  $X_D$  and  $s_D$  is the average and standard deviation of the differences

The degree of freedom is n-1

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### Z test

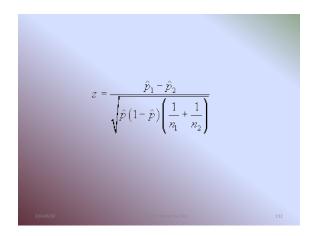
- Compares proportions from two independent samples
- Based on sampling distribution of the difference of proportions
- Allow calculation of differences and a confidence interval for the difference
- Is equivalent to chi-squared test
- Can be calculated by formula and statistical program

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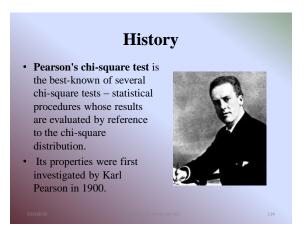
### Z test

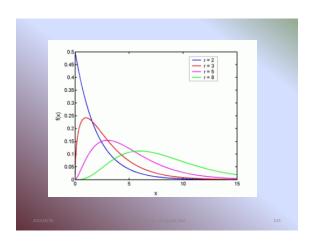
- Null hypothesis
  - -Two samples come from populations with the same proportions
- Assumption of the test
  - -Binary data
  - Sample is large. Both subjects with or without characteristics should be greater than 5 in both groups.

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### **Chi-squared test**

- Tests for association between two categorical variables
- When each variables has only 2 categories, it is equivalent to z test
- Based on the chi-squared distribution with n degree of freedom
- n = (no. of row 1) x (no. of column 1)
- It gives p value but not direct estimate or confidence interval as z test

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### **Chi-squared test**

- · Rationale of test
  - -Calculates the frequencies that would be expected if there was no association
  - It compares the observed frequencies and expected values
  - It they are very different, this provides evidence that there is an association
  - The test uses a formula based on chisquared distribution to give p value

### **Chi-squared test**

- · Null hypothesis
  - There is no association between the two variables in the population form which the samples come
- · Assumptions of test
  - -Large sample size
  - -At least 80% of expected frequencies must be greater than 5

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### **Chi-squared test**

- · If assumption do not hold
  - -Collapsing the table
  - -Continuity correction (Yates' correction)
  - -Fisher's exact test
- · Doing chi-squared test
  - Always use with frequencies, never use percentage
  - The formula works with all size tables
  - -Can be done by computer program

 $X^{2} = \sum_{i=1}^{n} \frac{(O_{i} - E_{i})^{2}}{E_{i}}$ 

where

 $X^2$  = Pearson's cumulative test statistic, which asymptotically approaches a  $\chi^2$  distribution.

 $O_i$  = an observed frequency;

 $E_i$  = an expected (theoretical) frequency, asserted by the null hypothesis;

n = the number of cells in the table.

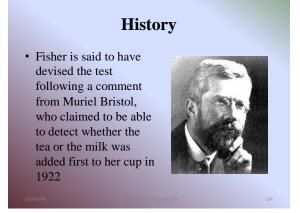
### Yates' Correction

- Chi-squared test based on frequencies (discrete) whilst the chi-squared distribution is continuous.
- The fit is not good in small sample size
- Yates' correction modified the chi-squared formula to make better fit
- Corrected p value (slightly bigger) should be reported

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$$\chi^2_{\rm Yates} = \sum_{i=1}^N \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$
 where: 
$$O_i = \text{an observed frequency}$$
 
$$E_i = \text{an expected (theoretical) frequency, asserted by the null hypothesis}$$
 
$$N = \text{number of distinct events}$$





### Fisher's Exact test

- Useful for small samples where chi-squared test is invalid
- Tests for an association between 2 categorical variables
- Normally used for 2 x 2 tables, but computer program allow for bigger tables
- Evaluating the probabilities associated with all possible tables which have the same row and column totals as the observed data, assuming the null hypothesis is true

Fisher's Exact test

- Based on exact probabilities, it is computationally intensive and may be slow or fail for large sample size.
- Give p values but not direct estimate or confidence interval

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### Fisher's Exact test

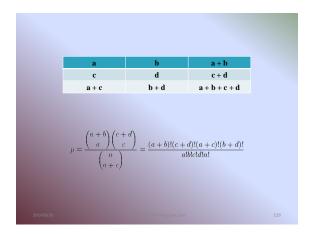
- · Null hypothesis
  - No association between the two variables in the population from which the samples
  - -Same null hypothesis as the chi-squared test
- · Assumptions of test
  - -none

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### Fisher's exact test

- Always use with frequencies, never use percentages for calculation
- No simple formula, statistical program needed
- Unless with good reason, use the two-sided p value
- It gives p values at least as big as the chi-squared test. For large sample size, p values are similar
- If in doubt about the sample size, use Fisher's exact test instead of chi-squared test.

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### Pearson's correlation

- It investigate the strength of a linear relationship between two continuous variables
- It is used when neither variable can be assumed to predict the other
- It gives an estimate, the correlation coefficient and a p value
- · A confidence interval can be calculated

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### Pearson's correlation

- Assumption
  - The relationship is linear
  - Normal distribution
    - For significant test at least one variable to be normally disturbed
    - For confidence intervals both variables should be normally distributed
  - A random sample within the range of interest

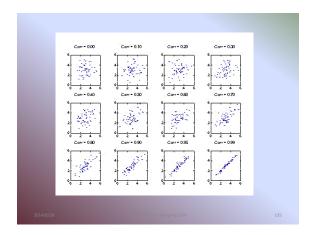
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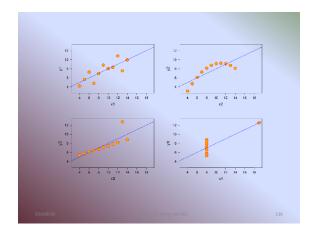
$$r = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}}.$$

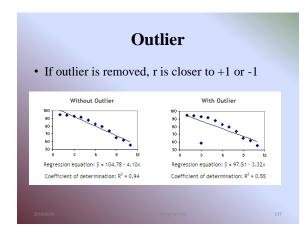
### Interpretation of r

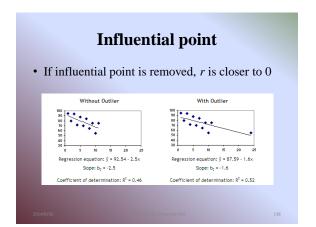
- *r* tells us how close is the linear relationship between two variables
- It lies between +1 and -1
- Negative (positive) values indicate negative (positive) linear relationship
- r = 0 indicate that is no linear relationship
- The closer the value +1 or -1, the stronger relationship between two variables

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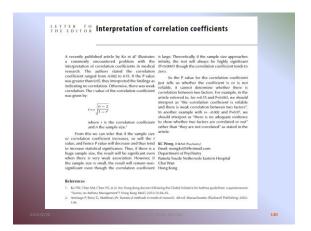




### Test and estimate of r

- A significant test can be done with null hypothesis that r = 0
- A confidence interval of r can be calculated
- Statistical significance of *r* directly related to sample size
  - If sample size is large, it may be statistically significant even the relationship is weak

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### Spearman's correlation

- It is used when none of the 2 variables follows a normal distribution – it is assumption for Pearson's correlation.
- · Null hypothesis
  - There is no tendency for one variable either to increase or to decrease as the other increases
- Assumption
  - The variables can be ranked
  - Monotonic relationship between 2 variables

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### Spearman's correlation

- This is calculated using same formula as for Pearson's correlation but uses the ranks of the data rather than the data values themselves
- It gives a values between -1 and +1
- p value can be obtained from statistical program

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### MANN WHITNEY U TEST (WILCOXON TWO SAMPLE SIGNED RANK TEST)

### **History**

- It is also called as Wilcoxon two sample signed ranked test.
- It was proposed initially by Frank Wilcoxon in 1945, for equal sample sizes, and extended to arbitrary sample sizes and in other ways by Henry Mann and his student Donald Ransom Whitney in 1947

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### Mann-Whitney U test

- · It is based on the rank of the data
- It gives p value but no estimate
- Given a table of cut-offs, the test is easy to do by hand
- If the sample is very small (both smaller than four observations), then statistical significant is impossible.

### Mann-Whitney U test

- The samples should be unrelated (independent)
- The test can be applied to ordinal data
- The population distributions should have the same shape (it tests the whole distribution)
- Tied values in the data
  - If number of ties is small, it still satisfactory
  - If data are ordinal and fall into only 3 or 4 categories, it is better to use chi-squared test

### **WILCOXON MATCHED PAIRS TEST**

### **History**

- It also called Wilcoxon signed rank test
- The test is named for Frank Wilcoxon (1892–1965) who, in a single paper, proposed both it and the ranksum test for two independent samples (Wilcoxon, 1945)



### Wilcoxon matched pairs test

- · Based on the signs of the differences in the pairs and the relative sizes of differences rather than the actual values
- It gives p values but no estimate
- Given a table of cut-offs, the test is easy to do by hand
- If the sample is smaller than 6, then statistically significance is impossible

### Wilcoxon matched pairs test

- · The data should be paired
- · The data should consist of numerical measurements (interval data)
- The distribution of the differences should be symmetrical. It is difficult to know in practice whether the distribution is symmetrical or not, but we can apply it with small sample size
- The zero difference (tie) should be omitted



### **Simple Linear Regression**

- Estimate the nature of linear relationship between two continuous variables
  - Dependent (response) variable
  - Independent (explanatory) variables
- The calculation based on least squared method – It minimize the sum of the squares of these residual ( = observed value – fitted values)

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### **Simple Linear Regression**

• Slope or regression coefficient is given by

$$\begin{split} \hat{\beta} &= \frac{\sum_{i=1}^{n} (x_{i} - \bar{x})(y_{i} - \bar{y})}{\sum_{i=1}^{n} (x_{i} - \bar{x})^{2}} = \frac{\sum_{i=1}^{n} x_{i} y_{i} - \frac{1}{n} \sum_{i=1}^{n} x_{i} \sum_{j=1}^{n} y_{j}}{\sum_{i=1}^{n} (x_{i}^{2}) - \frac{1}{n} (\sum_{i=1}^{n} x_{i})^{2}} \\ &= \frac{\bar{x} \bar{y} - \bar{x} \bar{y}}{\bar{x}^{2} - \bar{x}^{2}} = \frac{\text{Cov}[x, y]}{\text{Var}[x]} = r_{xy} \frac{s_{y}}{s_{x}}, \end{split}$$

- The line goes through the mean point: (x, y)
- Therefore the intercept is given by: a = y bx

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### **Interpretation of the equation**

- Regression coefficient gives the change in the outcome (y) for a unit change in the predictor variable (x)
- The intercept gives the value of y when x is 0
- The line gives the mean or expected value of *y* for each value of *x*

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### **Simple Linear Regression**

- If there is no relationship between *x* and *y*, the true regression coefficient *b* will be 0
- Can be tested using a form of t test
- The regression coefficient *b* is a useful summary to show how the two variables related
- 95% confidence interval can be calculated for b
- The equation of the line can be used for prediction

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### **Simple Linear Regression**

- Assumption
  - The relationship is linear
  - The distribution of the residuals is normal
  - The variance of the outcome y is constant over x
- Predication
  - Within sample prediction
  - Prediction outside the sample

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# Multivariate Analysis

### **Multivariate?**

- When there is more than one predictor variable on predicting the outcome variable
- E.g., head size (predictor) and recall (criterion)
- Recall Performance (in words) = 0.32 (head size in cm)
   + 1.67
- ...but how about other variables, such as IQ?
- · Linear combination of variables

$$Y' = a + b_1(x_1) + b_2(x_2) + \dots + b_k(x_k)$$

k is number of predictor variables

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### **Multivariate Statistics: the basics**

- To examine the relationship between variables, e.g., multiple regression
  - The relationship between predictor variables (can be discrete—e.g., gender, or continuous—e.g., income) and a continuous outcome variable
  - Predicting the performance in a certain task (e.g., memory) based on subjects' performance in other tasks (e.g., attention) and group status (e.g., showing hypertension)

### **Multivariate Statistics: the basics**

- Costs
  - Requires larger sample sizes
- Benefits
  - Increases precision using multiple measures of a construct
  - Including more variables to capture genuine relationships in the multidimensional and multicausal real world

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### **Multivariate Statistics: Data screening**

- · Relationships with other variables
  - <u>Multicollinearity</u> strong inter-correlations among predictor variables (|r| > 0.8)
    - · Observed in inter-correlation matrices
    - Redundancy among the predictor variables
  - − Tolerance 1 − R<sup>2</sup> ("Collinearity Statistics")
    - Larger = Better
    - If < 0.2 multicollinear

**Multivariate Statistics: Data screening** 

- How to tackle multicollinearity problems?
  - To combine closely related variables into a composite variable (e.g., averaging)—this requires some theoretical justifications
  - To use centering: transforming the closely related variables by subtracting the mean (averaged across all samples) from each sample

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## Multiple Regression

### **Multiple Regression**

- To assess the relationship between <u>one continuous</u> outcome variable (Y) and several discrete/or continuous predictor variables (X)
  - The best prediction of a outcome variable (Y') from the combination of predictor variables
  - $Y' = a + b_1(x_1) + b_2(x_2) + ... + b_k(x_k)$
  - b = unstandardized regression coefficient of each predictor variable
    - . Show how much Y' would change with a one unit increase in x
  - Regression analyses—to come up with b values that make Y (data) and Y' (prediction) as close as possible

**Multiple Regression: Type** 

- Standard
  - All variables are simultaneously entered into the regression equation
  - The regression coefficient of each predictor is estimated while {holding constant/ partialling out/ controlling for} the other predictor variables

### **Multiple Regression: Type**

- Sequential (block entry or hierarchical)
  - Predictor variables are entered in two or more steps
  - To evaluate the relationships between a <u>subset</u> of predictor variables and an outcome variable, after controlling for the effects of other subsets of predictor variables on the outcome variable
    - · Based on theories
  - Determine the significance of the change in  $R^2$  at each step to see if each newly added predictor makes a significant improvement in the predictive power of the regression equation

**Multiple Regression: Example 1** 

JobSatisfaction = 2.411 + (0.512)(SocialSupport) + (-0.109)(Stress) + (0.106)(Income)

- If all other variables are held constant:
  - Predicted job satisfaction increases by 0.512 for the increase of an unit of social support
  - Predicted job satisfaction decreases by 0.109 for the increase of an unit of stress
  - Predicted job satisfaction increases by 0.106 for the increase of an unit of income

### **Multiple Regression: Example 1**

- · How important are our predictor variables?
  - Regression coefficients and tests of significance (ttest)—If the coefficient is not significantly different from zero, that predictor will serve no use
- Common mistake: relative sizes of  $b_i$  reflect the relative importance of the predictors
  - E.g., Income is more important than Social Support because b is larger?
  - $\label{eq:control_control} JobSatisfaction = 2.411 + (0.512)(SocialSupport) + \\ (-0.109)(Stress) + (0.106)(Income)$

### **Multiple Regression: Example 1**

- When considering relative magnitudes of the coefficients, we should take into account the SD of the predictor variables
  - Solving the regression using standardized variables (beta)

		Unstandardize	d Coefficients	Standardized Coefficients		
Model		В	Std. Error	Beta	1	Sig.
1	(Constant)	2.411	.328		7.358	.000
	social support at work	.512	.057	.502	9.019	.000
	degree of stress at work	-109	.053	-115	-2.053	.041
	monthly income at work	.106	.050	.102	2.112	.036

 $\label{eq:constraint} \begin{aligned} \textit{JobSatisfaction}_{\textit{Z}} &= (0.502)(\textit{SocialSupport}_{\textit{Z}}) + \\ (-0.115)(\textit{Stress}_{\textit{Z}}) &+ (0.102)(\textit{Income}_{\textit{Z}}) \end{aligned}$ 

### **Multiple Regression: Example 1**

 $Y' = a + b_1(x_1) + b_2(x_2) + ... + b_k(x_k)$ 

- Common mistake: X₁ and Y are strongly correlated → b₁ has to be a significant regression coefficient in multiple regression models
  - A test of a predictor variable is done in the context of all other variables in the regression equation
  - After X<sub>2</sub>, X<sub>3</sub>, X<sub>4</sub>, ... and X<sub>k</sub> are partialled out (or held constant),
     X<sub>1</sub> may no longer be useful for predicting the Y
  - $b_1$  is coefficient for regression of Y on  $X_1$  when we partial out the effect of  $X_2, \ldots,$  and  $X_k$
  - E.g., math exam score and students' quantitative IQ

### Multiple Regression: Points to note

- <u>Sample size</u> considerations (rules of thumb)
  - -50 + 8 \* (number of predictors), or
  - Number of predictor variables should be smaller than (the number of subjects divided by 10)
- Limitations
  - Predictability does not tell us anything about the direction of causality in regression analyses
    - · Similar to "Correlation does not mean causation"
  - Regression assumes no measurement error
    - Must ensure **reliability** and validity of the variables

### Multiple Regression: Points to note

- Should all *available* variables be entered in the regression models?
  - Adding more predictor variables can change the betas of all other predictors
  - Excludes relevant variables (e.g., intelligence) → estimation of coefficients for the remaining variables may not reflect the whole pictures
  - Includes irrelevant variables (e.g., preference to ice-cream flavor in salary and success in universities) → increase the error variance and weaken the statistical power of the model

### **Multiple Regression: Points to note**

- Assumptions
  - Absence of multicollinearity among predictor variables
  - Normality, linearity, and homoscedasticity
    - If the underlying function between one or more of the predictors and the criterion is *not* linear, then the betas will be biased and unreliable, so it is important to look at all bivariate plots prior to the analyses

**Logistic Regression** 

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### **Logistic Regression**

- Can be used when the outcome (i.e., Y) is dichotomous
- Examples:
  - Does a sample have early-stage Alzheimer's disease?
  - Does a cancer patient respond to therapy?
  - Does a secondary school student smoke?
- · Compare to continuous outcomes
  - Global cognitive function
  - Tumor size
  - Packs/week

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### **Logistic Regression**

- To predict the membership in discrete groups (outcome variable, e.g., binary) from several predictor variables (which can be discrete or continuous)
  - E.g., predicting the occurrence of dementia (healthy old adults vs. earliest-stage Alzheimer's diseases)
     based on subjects' performance in cognitive tests
  - Binary outcome variable is most common, code reference group as 0 and occurrence group as 1 (useful for odd ratio interpretation)

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### **Logistic Regression**

 Goal = to find the best <u>linear</u> combination of predictors to maximize the likelihood of obtaining the observed outcome frequencies

$$ln\Big(\frac{p}{1-p}\Big) = b_0 + b_1 x$$

- p is the probability that the event Y occurs, p(Y=1)
- p/(1-p) is the *odds ratio*
- $-\ln[p/(1-p)]$  is the log odds ratio, or *logit*
- Can be standard or sequential

**Logistic Regression: Points to note** 

- Assumptions about variable distributions are not required, but may have more power if there are multivariate normality, and linearity among predictors
- Problems may occur if too few cases relative to predictor variables
- Absence of <u>multicollinearity</u> across predictor variables

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OTHERS

### **Bonferroni Correction**

- It is a simple method to correct the cut-off for statistical significant for multiple testing
- For  $\alpha = 0.05$ , if the test repeated for 20 times, there should be 1 false positive
- If there is *n* repeated testing, we need  $(1 \alpha)^n = 0.95$ , which approximately equal to  $1 n\alpha$ . So,  $\alpha$  is approximately equal to 0.05 / n
- Disadvantage too conservative

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### **Estimate for test of proportion**

- Risk difference  $(p_1 p_2)$ 
  - Use if actual size of difference is of interest
  - Most straight forward and useful for survey
- Relative risk  $(p_1/p_2)$ 
  - Use if relative risk is of interest
  - Useful when comparing the size of effect for several factor
  - Easier to interpret
  - Do not use for case control test

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### **Estimate for test of proportion**

- Odds ratio (ad / bc where a, b, c and d are 2x2 tables frequencies)
  - Use for case-control studies
  - Approximately equal to relative risk when the outcome is rare
  - Can be misinterpreted when the outcome is common
  - Can adjust for others factors using logistics regression

### **Others**

- · Analysis of variance
- Poisson Regression
- Kappa (inter-rater reliability)
- · McNamer test
- Epidemiology
- · Principle component analysis
- ......

### **Confidence Interval**

In statistics, a confidence interval (CI) is a
 particular kind of interval estimate of a population
 parameter and is used to indicate the reliability of
 an estimate. It is an observed interval, in principle
 different from sample to sample, that frequently
 includes the parameter of interest, if the
 experiment is repeated. How frequently the
 observed interval contains the parameter is
 determined by the confidence level or confidence
 coefficient.

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COMMON STATISTICAL SOFTWARE

### Choosing a package

- Cost
- Support
- · Your institution
- Usability
- Data managementType of analysis
- Specialized method
- Graphics

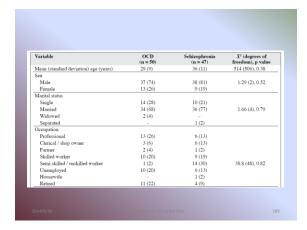
- · Size of datasets
- Transferring between packages
- · Testing
- · Operating system
- License versus perpetual copy
- Upgrades
- · Discounted versions

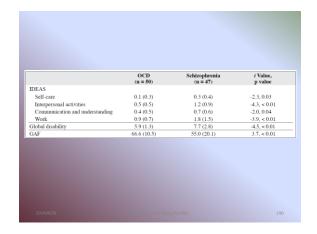
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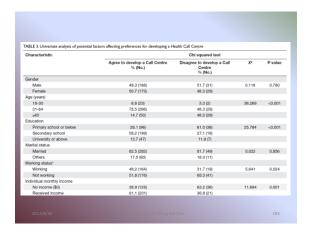




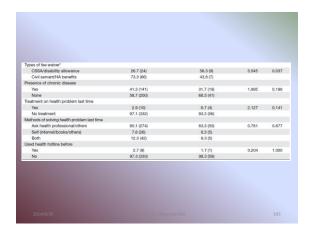


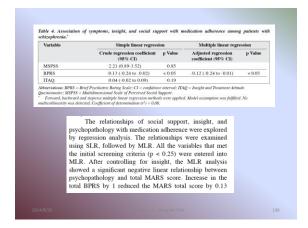
	Adults (< 65 years) [n = 7]	Elderly (≥ 65 years) [n = 12]	Chi-square	p Value
Psychiatric diagnoses				
Organic brain syndrome	2 (29%)	0	3.83	0.05
Dementia	0	8 (67%)	8.06	0.01
Mental retardation	4 (57%)	0	8.69	0.003
Schizophrenia	3 (43%)	4 (33%)	0.17	0.68
Delusional disorder	0	2 (17%)	1.30	0.25
Depression	1 (14%)	1 (8%)	0.17	0.68
Bipolar affective disorder	2 (29%)	0	3.83	0.05
Physical diagnoses				
Neurological	4 (57%)	4 (33%)	1.03	0.31
Gastro-intestinal / hepatic	2 (29%)	1 (8%)	1.36	0.24
Orthopaedic	1 (14%)	2 (17%)	0.02	0.89
Respiratory	0	2 (17%)	1.30	0.25
Endocrine	1 (14%)	2 (17%)	0.02	0.89
Urological	0	1 (8%)	0.62	0.43
Sensory	0	1 (8%)	0.62	0.43
Cardiovascular	1 (14%)	2 (17%)	0.02	0.89
Treatment factors				
Extrapyramidal symptom	3 (43%)	1 (8%)	3.17	0.08
Typical antipsychotics	5 (71%)	10 (83%)	0.38	0.54
Atypical antipsychotics	4 (57%)	1 (8%)	5.43	0.02
Anticonvulsants	4 (57%)	4 (33%)	1.03	0.31
Benzodiazepines	6 (86%)	1 (8%)	11.38	0.001
Anticholinergies	5 (71%)	6 (50%)	0.83	0.36
Antidepressants	1 (14%)	2 (17%)	0.02	0.89

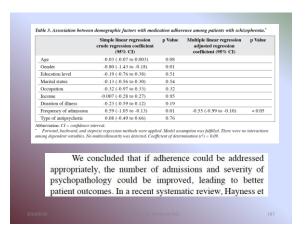
Depression 0.25 $p < 0.01$ Hopelessness 0.21 $p < 0.01$	Variable	Pearson's correlation (r)	Significance
Hopelessness 0.21 $p < 0.01$ Risk rescue score 0.13 $p < 0.05$ tent than those without morbidity (Table 1). There was	Age	0.12	p < 0.05
Risk rescue score 0.13 $p < 0.05$ tent than those without morbidity (Table 1). There was	Depression	0.25	p < 0.01
tent than those without morbidity (Table 1). There wa	Hopelessness	0.21	p < 0.01
• • • • • • • • • • • • • • • • • • • •	Risk rescue score	0.13	p < 0.05
e, hopelessness, depression, and lethality of the atter			













### You should learn...

- · Basic concept of hypothesis test
- Interpretation of the test result
- · Sampling method
- Identify and avoid common mistake in medical statistics
- At least, you should know when, how and where to seek advice

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- Oxford Handbook of Medical Statistics JL Peacock, PJ Peacock. Oxford University Press. 2011
- Statistical Methods in Medical Research 4th edition. P Armitage, G Berry, LNS Matthews. Blackwell Publishing. 2002
- Elementary Statistics Tables. HR Neave. Routledge. 1989
- http://www.cct.cuhk.edu.hk/stat/
  - Sample size estimation (CUHK)
  - http://udel.edu/~mcdonald/statintro.html
    - Handbook of Biological Statistics
- http://www.york.ac.uk/depts/maths/histstat/
  - History of Statistics (The University of York)

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### To sum up

- Now, you may forget half of the content
- You will forget 90% of the talk within 1 week
- · Learn it and revise it if you use it
- Consult statistician if necessary (better at the beginning of the study)
- Compared with mathematics, medical statistics is a "kid".
- My e-mail: kc@drkcwong.com

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### Thank You